1 More on disjunction

1.1 ‘Distributivity’ inferences

(1) Every student either solved Problem #1 or Problem #2
    ~Some students solved Problem #1 but not Problem #2 and some students solved
    Problem #1 and did not solve Problem #2.

This inference is not a logical entailment of (1), since (1) would be true, in terms of its literal
meaning, if, say, every student solved Problem #1 and no student solved Problem #2.

‘No! - test’

(2) Every student either solved Problem #1 or Problem #2
    a. No! No student solved Problem #1
    b. No! Every student solved Problem #1.

NB:

(3) a. Jack solved some of the problems
    b. No! he solved all of the problems

1.2 The distributivity inference is not predicted so far

Let us represent (1) as in (4) for short:

(4) \( \forall x (P_1(x) \lor P_2(x)) \)

According to the neo-Gricean approach, the scalar alternatives for (4) are as follows:
(5) \( \text{ALT}((4)) = \{ \forall x (P_1(x) \text{ or } P_2(x)), \forall x (P_1(x) \text{ and } P_2(x)) \} \)

Prediction: (4)’s strengthened meaning is as follows:

(6) a. \( \forall x (P_1(x) \text{ or } P_2(x)) \land \neg \forall x (P_1(x) \text{ and } P_2(x)) \)
   b. Every student solved Pb1 or Pb2, and not all of them solved both

But this is true in a situation where every student solved Pb1 and did not solve Pb2.

### 1.3 Refined alternatives for disjunction

To account for the observed distributivity inference, let us expand the set of alternatives for (4):

(7) \( \text{ALT}((4)) = \{ \forall x (P_1(x) \text{ or } P_2(x)), \forall x P_1(x), \forall x P_2(x), \forall x (P_1(x) \text{ and } P_2(x)) \} \)

Now, notice that both \( \forall x P_1(x) \) and \( \forall x P_2(x) \) a-entails (4). Hence the following additional SIs are predicted:

(8) a. \( \neg \forall x P_1(x) \)
   b. \( \neg \forall x P_2(x) \)

Equivalently:

(9) a. \( \exists x \neg P_1(x) \)
   b. \( \exists x \neg P_2(x) \)

The resulting reading is then:

(10) a. Every student solved Pb1 or Pb2, and some students did not solve Pb1 and some students did not solve Pb2
   b. Every student solved Pb1 or Pb2, and some students solved Pb1 but not Pb2, and some students solved Pb2 but not Pb1

How should we modify Horn’s scale for disjunction? Sauerland (2004) and other’s proposal:

(11) For any sentence \( S \) in which a disjunctive phrase \( A \text{ or } B \) occurs:
   a. \( \text{ALT}(\{s \ldots (A \text{ or } B) \ldots \}) = \text{ALT}(\{s’ \ldots A \ldots \}) \cup \text{ALT}(\{s” \ldots B \ldots \}) \)
      i.e.:
   b. The alternatives for a sentence of the form \( [s \ldots (A \text{ or } B) \ldots ] \) include all the alternatives of \( [s’ \ldots A \ldots ] \) and of \( [s” \ldots B \ldots ] \)

Sauerland’s (2004) specific proposal:

(12) a. Scale for disjunction: \( \{ \text{or, and, L, R} \} \), with
   b. \( (A \text{ L } B) \sim A, (A \text{ R } B) \sim B \).
1.4 Refined alternatives for conjunction

The ‘distributivity’ inference can also be observed with conjunction

(13) No student solved both Pb1 and Pb2
    \[ \therefore \text{Some students solved Pb1 (and not Pb2), some students solved Pb2 (and not Pb1)} \]

(14) a. \(\neg \exists x (P_1(x) \text{ and } P_2(x))\)
    b. \(\forall x (\neg P_1(x) \text{ and } P_2(x))\)
    c. \(\forall x (\neg P_1(x) \text{ or } \neg P_2(x))\)

So the effect is entirely similar. That is, if we had started with a syntactic form like (14-c), we would have expected the following distributivity inferences:

(15) a. \(\exists x (\neg P_1(x) \text{ and } P_2(x))\)
    b. \(\exists x (P_2(x) \text{ and } \neg P_1(x))\)

We can predict this result by assuming that the alternatives induced by conjunction are exactly the same as those induced by disjunction. In this case we have:

(16) \(\text{ALT}((14-a)) = \{\neg \exists x (P_1(x) \text{ or } P_2(x)), \neg \exists x P_1(x), \neg \exists x P_2(x), \neg \exists x (P_1(x) \text{ and } P_2(x))\}\)

We then get the following SIs:

(17) a. \(\neg (\neg \exists x P_1(x))\)
    b. \(\neg (\neg \exists x P_2(x))\)

i.e.:

(18) a. \(\exists x P_1(x)\)
    b. \(\exists x P_2(x)\)

1.5 Distributivity inferences with necessity modals: ‘free-choice’ effects

(19) The students are required to either solve Pb1 or Pb2 \(\therefore\) The students are not required to solve both, and they are free to choose which one to choose.

Strengthened meaning:

(20) The students are required to solve Pb1 or Pb2, and they are allowed to solve either one without solving the other

Schematic representation for (19) (‘\(\Box\)’ stands for necessity modals, ‘\(\Diamond\)’ for possibility modals):

(21) \(\Box (A \text{ or } B)\)
(22) \(\text{ALT}((21)) = \{\Box (A \text{ or } B), \Box A, \Box B, \Box (A \text{ and } B)\}\)
Note that ‘□’ can be viewed as a kind of \textit{universal quantifier}, namely a universal quantifier over \textit{possible worlds}:

\begin{equation}
\Box A \rightarrow \text{In all the worlds in which what is required to be the case is in fact the case, A is true.}
\end{equation}

Given that both ‘□A’ and ‘□B’ a-entails (21), we predict the following ‘strengthened’ meaning:

\begin{equation}
\begin{align*}
a. & \quad \Box(A \lor B) \land \neg \Box A \land \neg \Box B \land \neg \Box(A \land B) \\
b. & \quad \Box(A \lor B) \land \Diamond \neg A \land \Diamond \neg B \\
c. & \quad \text{The students are required to solve Pb1 or Pb2, they are allowed not to solve Pb1 and they are allowed not to solve Pb2.}
\end{align*}
\end{equation}

\section{Symmetry strikes again!}

Let us go back to simple disjunctions:

\begin{equation}
A \lor B
\end{equation}

Now the alternatives for (25) are as in (26):

\begin{equation}
\text{ALT}((25))=\{A \lor B, A, B, A \land B\}
\end{equation}

Note that both ‘A’ and ‘B’ a-entails (25). Hence it is predicted that (25)’s strengthened meaning is as follows:

\begin{equation}
(A \lor B) \land \neg A \land \neg B \land \neg (A \land B)
\end{equation}

But this is \textit{contradictory}!

\section{Another problem with disjunctions – Chierchia (2004)}

In the case of the initial symmetry problem, we dealt with it by restricting the set of alternatives (by eliminating one of the two ‘symmetric’ alternatives). This is not a viable option for our new case of symmetry. Furthermore, if we choose to ignore distributivity inferences and free-choice effects and go back to Horn’s original scale for disjunction, we encounter an other problem.

\subsection{Example #1: scalar items in the scope of disjunction}

\begin{equation}
\text{Context: Which problems did Jack solve?} \\
\hspace{1cm} \text{Jack solved some of the Maths problems or all of the Physics problems.}
\end{equation}

Alternatives of (28) (assuming now Horn’s scale for disjunction):
(29)  
   a. Jack solved either all of the Maths problems or all of the Physics problems.
   b. Jack solved some of the Maths problems and all of the Physics problems.
   c. (Allowing for multiple substitutions)
      Jack solved all of the Maths problems and all of the Physics problems.

All these alternatives a-entails (28). Hence it is predicted that (29) implicates their negation. Consider in particular the negation of (29-a):

(30) Not (Jack solved all of the Maths problems or incl Jack solved all of the physics problems)  

⇒ Jack didn’t solve all of the Physics problems

This is clearly much too strong. In normal contexts, one understands that the author of (28) is uncertain as to whether or not Jack solved all of the Physics problems.

What if we get rid of the offending alternative? We are left with the negations of the remaining alternatives, namely:

(31)  
   a. It is not the case that Jack solved both some of the Maths problems and all of the Physics problems.
   b. It is not the case that Jack solved both all of the Maths problems and all of the Physics problems.

(31-a) entails (31-b). Hence the only inference that is predicted is (31-a).

This is ok, but too weak. It is compatible with a situation where Jack solved all of the Maths problems and no Physics problems. But one tends to infer, from (28), that in any case Jack did not solve all of the Maths problems.

(32) Negative objections as tests for scalar implicatures  
Uttered as replies to (28):

   a. No! You are wrong! Jack solved ALL of the Maths problems.
   b. No! You are wrong! Jack solved both some maths problems and some physics problems.
   c. #No! You are wrong! Jack solved all of the physics problems.

This suggests that we would like to derive the following strengthened meaning for (28):

(33) Either Jack solved some of the Maths problems, did not solve all of them, and did not solve any Physics problems, or he solved all the Physics problems and did not solve any Maths problems.

2.2 Example #1: Multiple disjunctions

(34) Context: Who did Bill call yesterday?
a. He called either Jack, Peter or Sue
b. He called either Jack or Peter or Sue
c. A or B or C

a. A or B or C
b. A or (B or C)
c. (A or B) or C

Let us assume the latter.

(36) Either he called Jack or Peter, or he called Sue.

Alternatives:

(37) a. (A and B) or C
    b. (A or B) and C
    c. (A and B) and C

Pb: the negation of (37-a) entails ‘not C’.

Negations of (37-b) and (37-c):

(38) a. (Not A and Not B) or not C
    b. Not A or Not B or not C

A situation in which A and B are true but C is not is compatible both with the assertion and both inferences in (38). Yet, (34) suggests that Bill called only one of the three. This is further confirmed by the No!-test.

(39) As a reply to the answers in (34):
    a. No! You are wrong! He talked to BOTH Jack and Peter.
    b. No! You are wrong! He talked to BOTH Jack and Sue.
    c. No! You are wrong! He talked to BOTH Peter and Sue.

An aside on multiple exclusive disjunctions

The reading we want is NOT equivalent to what we would get with two exclusive disjunctions.

\((A \text{ or}_{\text{incl}} B) \text{ or}_{\text{incl}} C\) is true if and only if either exactly one or exactly three of the disjuncts are true.
Problem (optional):

Let $S$ be a sentence of the following form: $(p_n \, \text{or excl} \, (p_{n-1} \, \text{or excl} \, (\ldots \, \text{or excl} \, (p_2 \, \text{or excl} \, p_1)))))\ldots$).

Prove that $S$ is true if and only if the number of true disjuncts in $\{p_1, \ldots, p_n\}$ is an odd number.

3 A refined neo-Gricean account - Sauerland (2004); Spector (2003)

3.1 Ignorance implicatures

(40) Yesterday, Jack solved either $Pb_1$ or $Pb_2$

$\neg$The speaker does not know whether Jack solved $Pb_1$ or $Pb_2$ (cf. Gazdar 1979; who called such inferences 'clausal implicatures')


3.2.1 First step: primary implicatures

Notation: Let us write ‘$K\phi$’ for ‘the speaker believes that $\phi$ is true.’

Strictly speaking, what the maxim of quantity gives us is the following:

(41) a. If $S$ is uttered and $\phi$ belongs to $\text{ALT}(S)$, and $\phi$ a-entails $S$, then the speaker does not have the belief that $\phi$ is true.

i.e.

b. If $S$ is uttered and a member $\phi$ of $\text{ALT}(S)$ a-entails $S$, then $\neg K\phi$.

Let us call such inferences ‘Primary Implicatures’.
Note that ‘\(\neg K\phi\)’ (i.e. the speaker does not have the belief that \(\phi\) is true) is not the same as ‘\(K\neg\phi\)’ (i.e. the speaker believes that \(\phi\) is false).

**Application to simple disjunctive statements**

(42) A or B

Consequence of the maxim of quality: the author of (42) believes that ‘A or B’ is true

(43) K(A or B)

Primary implicatures for (42):

(44) a. \(\neg KA\)
b. \(\neg KB\)
c. \(\neg K(A\ and\ B)\)

Some consequences:

1. The speaker does not believe that A is false: if he did, given that he believes that A or B is true, he would have to believe that B is true, contrary to (44-b).

2. Symmetrically, the speaker does not believe that B is false.

In short:

(45) \((K(A\ or\ B) & \neg KA & \neg KB) \Rightarrow (\neg K\neg A & \neg K\neg B)\)

Hence the author of (42) has no opinion regarding the truth value of A, nor has he one regarding that of B.
3.2.2 Second step: secondary implicatures

General rule for deriving secondary implicatures: Whenever doing so is not contradictory given all that can be deduced from the first step, strengthen ‘¬Kφ’ into ‘K¬φ’ (this is the epistemic step)

More formally: Let S be a sentence, and φ an alternative of S. Then:
If ¬Kφ is a primary implicatures, and if K¬φ is consistent with all the primary implicatures together with KS, then K¬φ is a secondary implicature of S.

Illustration. Consider again a simple disjunctive statement:

(46) A or B
(47) Maxim of Quality:
    K(A or B)
(48) Primary implicatures for (46):
    a. ¬KA
    b. ¬KB
    c. ¬K(A and B)
(49) Further consequences:
    a. ¬K¬A
    b. ¬K¬B
(50) Secondary Implicatures for (46):
    a. K¬A (conflicts with (49-a))
    b. K¬B (conflicts with (49-b))
    c. ok: K¬(A or B)

Problem: Show that this procedure delivers right results for the cases discussed in section 2.

Back to distributivity inferences

(51) Every student solved Problem #1 or Problem #2
(52) Quality:
    K(every st. solved Pb1 or Pb2)
(53) Primary implicatures:
    a. ¬K(every st. solved Pb1)
    b. ¬K(every st. solved Pb2)
    c. ¬K(every st. solved Pb1 and Pb2)
Now, note that one can consistently believe (51) and simultaneously deny that every student solved Problem 1 and deny that every student solved Problem 2. In other words, it is possible to perform the epistemic step (moving ‘¬’ from the left of $K$ to its right) for each of the relevant alternative. So in the second step, we get:

(54) Secondary implicatures  
   a. $K\neg$(every st. solved Pb1)  
   b. $K\neg$(every st. solved Pb2)

**NB:** When the result of the initial neo-Gricean algorithm is not contradictory, it coincides with the result of Sauerland’s procedure. In practice, this means that we will have to care about the two-step procedures only in a restricted class of cases, all involving conjunction or disjunction.

4 SIs and exhaustivity – van Rooij and Schulz (2004); Spector (2003); Fox (2007)

4.1 Only  

(55) I have only solved some of the problems  
   $\sim$I have solved some but not all of the problems.  

(56) Jack only solved problem #1 or problem #2  
   $\sim$Jack solved one of the two problems but not both.

**Generalization:** Only [$S,...,t_F,...$,] where $t$ is a scalar item, expresses the conjunction of $S$ and $S$’s scalar implicatures.

Non-scalar cases:  

(57) Mary only talked to Jack

4.2 Textbook entry for only  

(58) Only  
   a. only takes two arguments: the linguistic expression E (called the *prejacent*) it combines with and a set of alternatives (expressions that could have been used instead of E)  
   b. Alternative sets are constrained by focus-marking  
   c. Only(ALT)(φ) = φ is true and every member of C is false except (57) itself.

In the case of (57), where focus is on ‘Jack’, ALT = {Mary talked to Jack, Mary talked to Sue, Mary talked to Alfred, ...}.  

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4.3 Refinement

More needs to be said: *only* should only exclude alternatives that are not entailed by its prejacent.

(59) Mary only talked to [Jack and Peter]₀

(60) ALT((59)) = Mary talked to Jack, Mary talked to Peter, Mary talked to Sue, Mary talked to Alfred and Max, . . .

*only* should not exclude ‘Mary talked to Jack’ nor ’Mary talked to Peter’.


(61) Only(ALT)(φ) = φ is true and every member of ALT if false except those entailed by φ

Second refinement

(62) Mary only talked to [Peter, Sue or Mary]₀ ⇒ Mary talked to only one of them, and
to nobody else

Here we need to give *only* a semantics that incorporates the insights of Sauerland (and others)’s analysis.

Intuitive idea: *Only(ALT)(φ)* should exclude the biggest possible subset of ALT such that is members can all be negated together ‘safely’, i.e. without getting us into a contradiction. For a formally explicit version of this idea see Fox (2007) (also relevant: van Rooij and Schulz (2004); Spector (2003)).

What we should remember : ‘Only(ALT)(φ)’ returns the conjunction of φ and of the negations of some of its alternatives, in a way that parallels the derivation of scalar implicatures for sentences without an *only*.

4.4 Exhaustive Readings

(63) a. Who did Jack talk to?
    b. Jack talked to Peter and Sue ⇒ Jack did not talk to anyone besides Peter and Sue

(64) Jack only talked to Peter and Sue

So even the 'non-scalar’ cases have a counterpart without *only*. There should be a Gricean account which simultaneously deals with scalar implicatures and exhaustivity effects of the type illustrated in (63) (see Spector (2003); van Rooij and Schulz (2004))
Exhaustivity operators: Maybe scalar implicatures and exhaustivity effects could be accounted for in terms of the optional presence of a covert ‘only’, a so-called exhaustivity operator . . . More on this tomorrow.

References


